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**TECHNICAL MEMORANDUM**

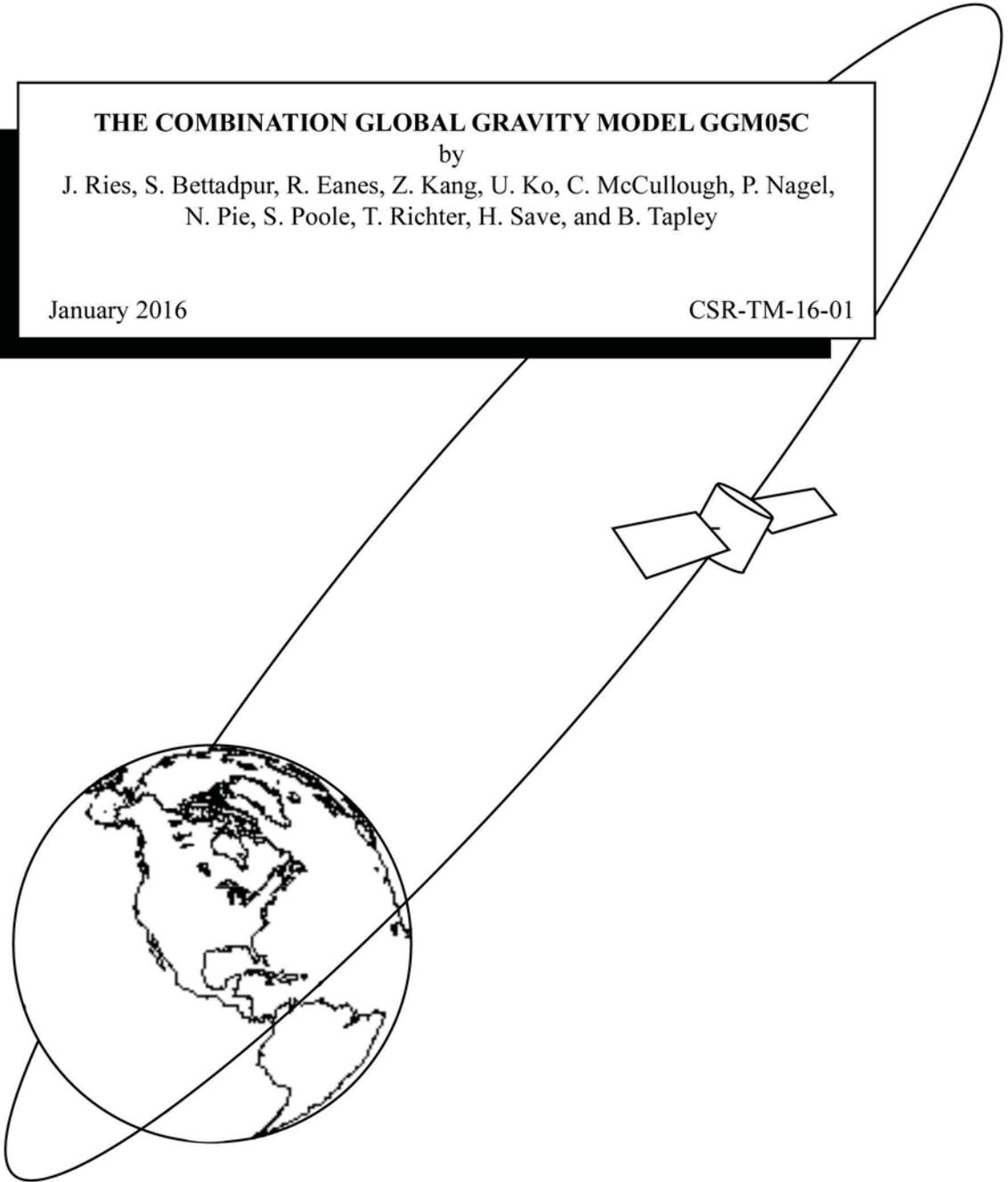
**THE COMBINATION GLOBAL GRAVITY MODEL GGM05C**

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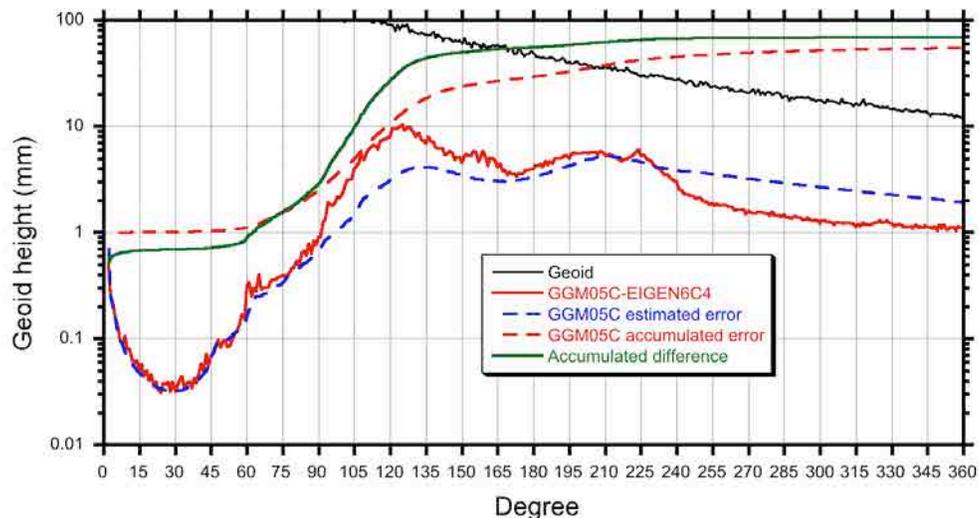
Byron D. Tapley

# The Combination Global Gravity Model GGM05C

The GGM05 model suite consists of three mean Earth gravity field models. GGM05S<sup>1</sup> was derived using only GRACE data (Tapley et al. 2013), and GGM05G<sup>2</sup> is derived from GRACE and GOCE data (Bettadpur et al., 2015). GGM05C, described in this document, combines GRACE, GOCE and terrestrial gravity data.

The GGM05C gravity model was estimated to spherical harmonic degree and order 360 from a combination of GRACE and GOCE gravity information (based on GGM05G) and surface gravity anomalies from DTU13 (Andersen et al., 2014). The 2 minute resolution anomalies were used, assuming that they were classical gravity anomalies (i.e., defined on the ellipsoid). The first step was a low pass filter applied to the DTU13 global anomaly field using the Driscoll and Healy algorithm (Driscoll and Healy, 1994). This was followed by a spherical harmonic analysis of the gravity anomaly set on the ellipsoid, where the coefficients were analytically transformed following the approach described in Claessens (2005). This was taken to degree 540, but only the coefficients up to degree 360 were used. Rather than reprocess the surface gravity data, the full covariance from GGM03C (Tapley et al., 2007) was adopted as apriori. The covariance was then modified so that, below degree 240, the terrestrial information was severely downweighted in order to preserve the accuracy of the GRACE and GOCE gravity contribution. This artificial covariance was used to combine the surface gravity information with GGM05G to obtain the GGM05C solution.

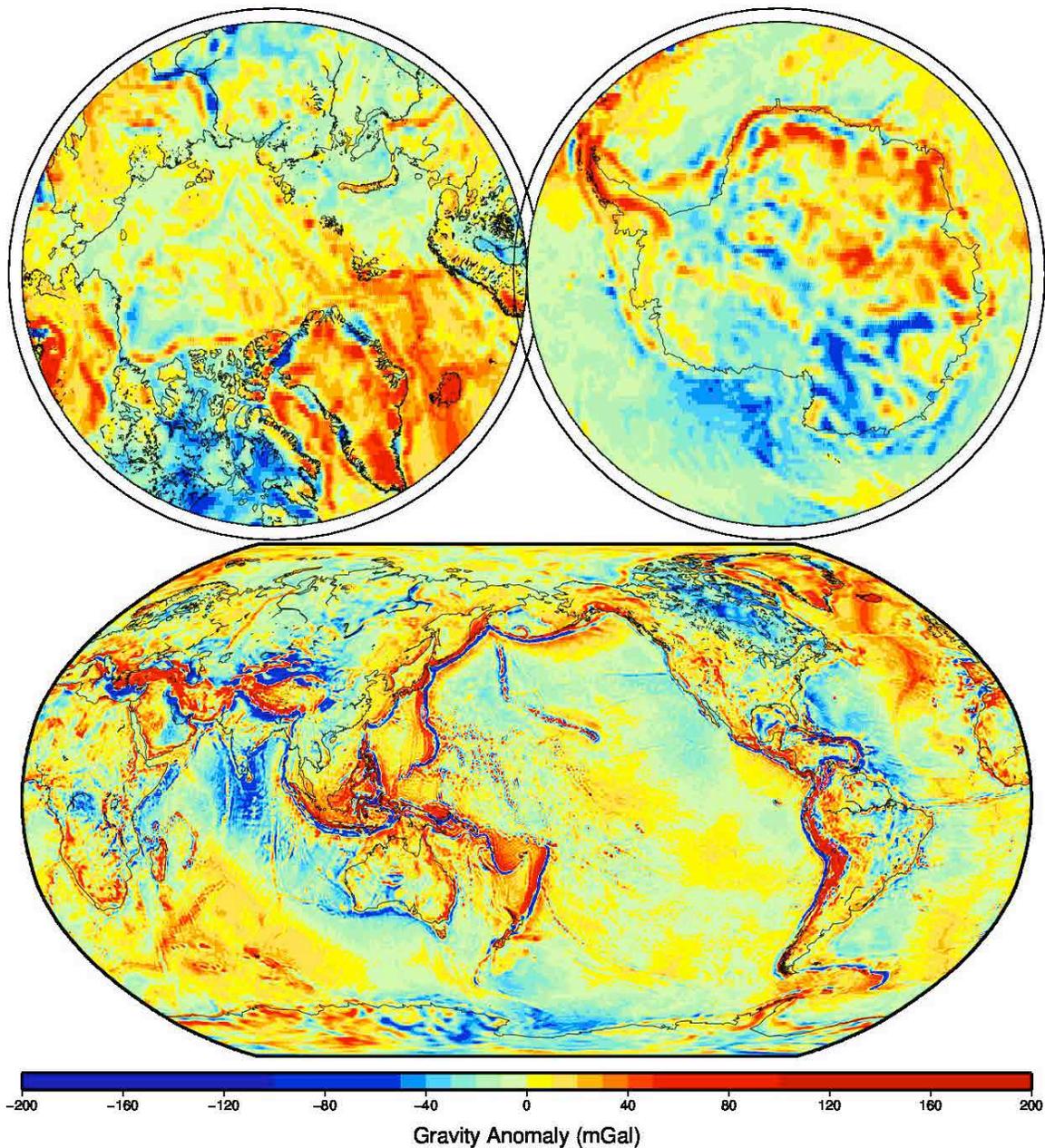
Figure 1 illustrates the degree error statistics of the GGM05C gravity field model in units of geoid height. The estimated errors, by degree and accumulated, are also compared to the degree difference variance with EIGEN6C4 (Förste et al., 2015). There is a significant difference in the mid-degrees, which is likely the result of the up-weighting of GOCE relative to GRACE adopted for GGM05G. Above degree 240, the differences are smaller than the estimated error, which is likely due to both models using very similar terrestrial gravity information. Figure 2 illustrates the gravity anomaly map from GGM05C computed to degree/order 360 with no smoothing.



**Figure 1.** Square root of the degree error variance for GGM05C compared to the square root of the degree difference variance with EIGEN6C4, shown in terms of geoid height.

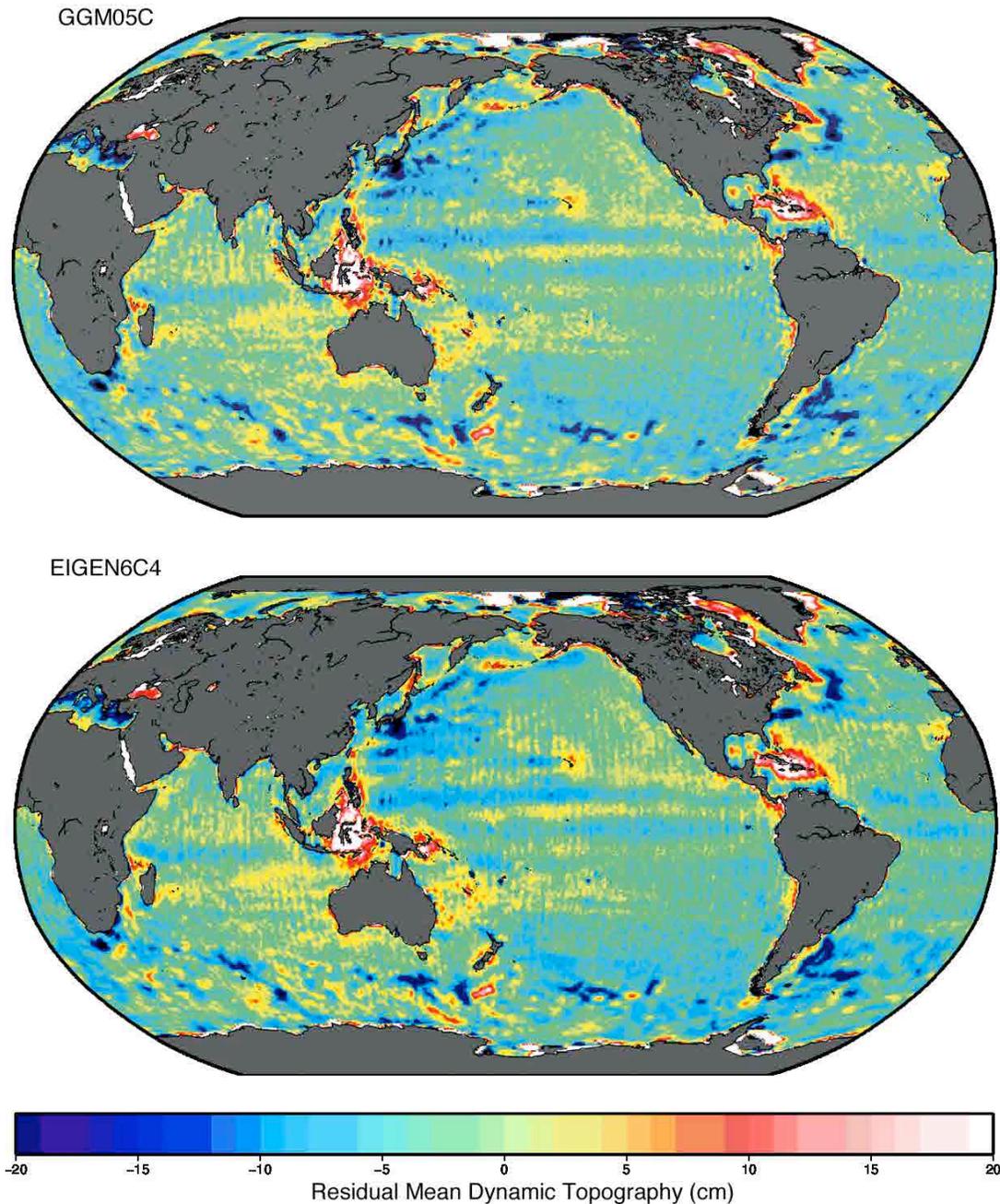
<sup>1</sup> [ftp://ftp.csr.utexas.edu/pub/grace/GGM05/README\\_GGM05S.pdf](ftp://ftp.csr.utexas.edu/pub/grace/GGM05/README_GGM05S.pdf)

<sup>2</sup> [ftp://ftp.csr.utexas.edu/pub/grace/GGM05/README\\_GGM05G.pdf](ftp://ftp.csr.utexas.edu/pub/grace/GGM05/README_GGM05G.pdf)



**Figure 2.** Gravity anomalies computed from GGM05c to degree and order 360 (no smoothing is required for GGM05C). Color table taken from the International Gravimetric Bureau ([http://bgi.omp.obs-mip.fr/content/download/1305/8531/file/WGM\\_2012\\_explanatory\\_leaflet.pdf](http://bgi.omp.obs-mip.fr/content/download/1305/8531/file/WGM_2012_explanatory_leaflet.pdf)).

To evaluate the geoid accuracy over the oceans, it is useful to compare the mean dynamic topography implied by the mean sea surface (in this case the mean sea surface is CLS01 [Hernandez and Schaeffer, 2001]) minus the geoid to some estimate of the mean dynamic topography (in this case CLS09 MDT [Rio et al., 2012]). This lets us look at the shorter wavelengths and explore for artifacts in the geoid model. Figure 3 shows such a comparison for GGM05C and EIGEN6C4. The features are similar, indicating that the residuals are driven largely by imperfections in the mean dynamic topography model rather than the geoid model. An artifact common to GRACE-based gravity models is the appearance of north-south ‘striations’ that are the result of increased errors in the resonant and near-sectorial harmonics. These terms are more susceptible to long-wavelength dynamical modeling errors, which may not be accurately reflected by the error covariance matrix. As described in the construction of GGM05G, the GOCE data was deliberately up-weighted with respect to GRACE, reducing the appearance of these striations in the MDT residuals.



**Figure 3.** Residual mean dynamic topography =  $CLS09 - (CLS01 - \text{geoid})$  for geoid models GGM05C (top) and EIGEN6C4 (bottom), computed to degree and order 240 and using a spherical distance of 360 km six-sigma for smoothing. EGM2008 was used to remove signal between degree 241 and 1080 to isolate the satellite contributions to the gravity models.

Using gravity data from the National Geospatial-Intelligence Agency (NGA, 2014), GGM05C is compared to several other global gravity models in Table 1 (all fields limited to degree and order 240 to highlight the satellite contributions). The areas selected for testing were chosen because terrestrial gravity data in these regions were not available for EGM2008 (Pavlis et al., 2012). No solution is best everywhere, but all show improvement over EGM2008. GGM05C shows identical performance to GGM05G, indicating that the addition of the DTU13 gravity data has not degraded the performance in these areas. Similar comparisons using GRAV-D data from NOAA’s National Geodetic Survey showed little discrimination between the models.

**Table 1.** Comparison of NGA gravity data, in terms difference variance (mgal<sup>2</sup>). A 0.5 degree radius Hanning filter has been applied (courtesy of T. Richter).

Gravity solution	Chile	Indonesia	Nepal
EGM2008	122	63.3	324
GGM05G (GRACE+GOCE)	112	60.3	224
GGM05C (GRACE+GOCE+surface gravity)	112	60.3	224
GOCO05S (GRACE+GOCE+regularization)	113	57.2	235
EIGEN6C4 (GRACE+GOCE+surface gravity)	102	58.3	250

**Additional Notes on the GGM05C gravity field solution and background modeling:**

GGM05C is provided as spherical harmonic coefficients. C00 is defined to be exactly 1, and the degree one terms are defined to be exactly 0. The epoch of GGM05C is 2008.0, the approximate midpoint of the ten-year GRACE data span used. No rates were applied in the background processing nor were any earthquakes removed. The solution is intended as a simple mean of the gravity field during the GRACE/GOCE observation period. Changes in the gravity field, such as due to major earthquakes, are better observed by combining monthly solutions before and after the event of interest.

C20 is a zero-tide value, *i.e.* it includes the zero-frequency (permanent) tide contribution; in order to convert to a tide-free system, add  $4.200 \times 10^{-9}$ . Rotational deformation (pole tide) was modeled using the IERS2010 conventions.

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## Appendix:

### Spherical harmonic coefficient description:

The coefficients for GGM05C are normalized according to the so-called “fully-normalized” convention, where the squared norm of a spherical harmonic over a unit sphere is  $4\pi$  (see below). The standard deviations or ‘sigmas’ (approximately calibrated, not the formal values) are included with the coefficients. The Earth radius and GM to be used for scaling in the expression for the geopotential are included in the coefficient file.

### Normalization convention:

If  $\varphi$  denotes the geographical latitude of a field point ( $0^\circ$  at equator,  $90^\circ$  at the North pole, and  $-90^\circ$  at the South pole), and if  $u = \sin \varphi$ , then the un-normalized Legendre Polynomial of degree  $l$  is defined by

$$P_l(u) = \frac{1}{2^l \times l!} \times \frac{d^l}{du^l} (u^2 - 1)^l$$

The definition of the un-normalized Associated Legendre Polynomial is then

$$P_{lm}(u) = (1 - u^2)^{\frac{m}{2}} \frac{d^m}{du^m} P_l(u)$$

If the normalization factor is defined such that

$$N_{lm}^2 = \frac{(2 - \delta_{0m})(2l + 1)(l - m)!}{(l + m)!}$$

and the Associated Legendre Polynomials are normalized by

$$\bar{P}_{lm} = N_{lm} P_{lm}$$

then, over a unit sphere S

$$\int_S \left[ \bar{P}_{lm}(\sin \varphi) \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} \right]^2 dS = 4\pi$$

In this convention, the relationship of the spherical harmonic coefficients to the mass distribution becomes

$$\begin{Bmatrix} \bar{C}_{lm} \\ \bar{S}_{lm} \end{Bmatrix} = \frac{1}{(2l + 1)M_e} \times \iiint_{Global} \left( \frac{r'}{a_e} \right)^l \bar{P}_{lm}(\sin \varphi') \begin{Bmatrix} \cos m\lambda' \\ \sin m\lambda' \end{Bmatrix} dM$$

where  $r'$ ,  $\varphi'$  and  $\lambda'$  are the coordinates of the mass element  $dM$  in the integrand. The integration is carried out over the entire mass envelope of the Earth system, including its solid and fluid components.

This convention is consistent with the definition of fully-normalized harmonics in NRC (1997), and textbooks such as Heiskanen and Moritz (1966) and Torge (1980).